Large deviations for multivalued backward stochastic differential equations

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1. Introduction
2. Multivalued BSDEs
3. Large deviations

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Preliminaries

Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables.

Law of Large Numbers

$\mathbb{E}(X_1) = \mu \in \mathbb{R}$, $Var(X_1) = \sigma^2 \in]0, +\infty[$.

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i;\ \left(\overline{X}_n\right)_n \longrightarrow \mu$$

$$\mu = 0,\ \Gamma = \{x : |x| \geq \alpha\};\ \mathbb{P}(\overline{X}_n \in \Gamma) \leq \frac{Var(\overline{X}_n)}{\alpha^2}$$

Large Deviation Principle (LDP)

Exponentially

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Example : LDP for i.i.d. sequences

Let $(X_n)_n$ be i.i.d. random variables with $\mathbb{P}(X_1 = 0) = \mathbb{P}(X_1 = 1) = \frac{1}{2}$.

$$\forall a > \frac{1}{2}, \ \lim_{n\to\infty} \frac{1}{n} \log \mathbb{P}(S_n \geq an) = -\Lambda(a)$$

$$\Lambda(x) \triangleq \begin{cases} \log 2 + x \log x + (1-x)\log(1-x), \ x \in [0,1] \\ \infty \end{cases}$$

Cramer (1938)

Let $(X_n)_n$ be i.i.d. $\mathbb{R}$-valued random variables satisfying $\mathbb{E}(e^{tX_1}) < \infty$, $\forall t \in \mathbb{R}$. Then, for all $a > \mathbb{E}(X_1)$,

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SDEs and BSDEs

$$X_t^{s,x,\varepsilon} = x + \int_s^t \beta(X_r^{s,x,\varepsilon})dr + \sqrt{\varepsilon}\int_s^t \sigma(X_r^{s,x,\varepsilon})dW_r, \; s \le t \le T$$

$$\varphi_t^{s,x} = x + \int_s^t \beta(\varphi_r^{s,x})dr, \; s \le t \le T$$

$$Y_t^{s,x,\varepsilon} = g(X_T^{s,x,\varepsilon}) + \int_t^T f(r, X_r^{s,x,\varepsilon}, Y_r^{s,x,\varepsilon}, Z_r^{s,x,\varepsilon})dr - \int_t^T Z_r^{s,x,\varepsilon}dW_r$$

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LDP for SDEs and BSDEs

Freidlin and Wentzell (1984)

$(X^{s,x,\varepsilon})_{\varepsilon>0}$ converges in probability, as ε goes to 0, to $(\varphi_t^{s,x})_{s\leq t\leq T}$ and satisfies a LDP.

Rainero (2006)

- $\varepsilon \in]0,1]$, $(s,x) \in [0,T] \times \mathcal{K}$, where $\mathcal{K}$ is a compact subset of $\mathbb{R}^d$. Then, there exists $C > 0$ independent of ε, s and x such that

$$\mathbb{E}\left(\sup_{s\leq t\leq T} |Y_t^{s,x,\varepsilon} - \psi_t^{s,x}|^2 + \int_s^T \|Z_r^{s,x,\varepsilon}\|^2 dr\right) \leq C\varepsilon,$$

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LDP for Reflected BSDEs

For every $s \leq t \leq T$,

$$\begin{cases} X_t^{s,x,\varepsilon} = x + \int_s^t \beta(X_r^{s,x,\varepsilon})dr + \sqrt{\varepsilon}\int_s^t \sigma(X_r^{s,x,\varepsilon})dW_r + \rho_t^{s,x,\varepsilon} - \rho_s^{s,x,\varepsilon} \\ \rho_t^{s,x,\varepsilon} = \int_0^t \nabla\phi(X_r^{s,x,\varepsilon})d \mid \rho^{s,x,\varepsilon} \mid_r, \\ \mid \rho^{s,x,\varepsilon} \mid_t = \int_0^t 1_{\{X_r^{s,x,\varepsilon} \in \partial\Theta\}} d \mid \rho^{s,x,\varepsilon} \mid_r \\ Y_t^{s,x,\varepsilon} = g(X_T^{s,x,\varepsilon}) + \int_t^T f(r, X_r^{s,x,\varepsilon}, Y_r^{s,x,\varepsilon}, Z_r^{s,x,\varepsilon})dr - \int_t^T Z_r^{s,x,\varepsilon} dW_r \\ - \int_t^T U_r^{s,x,\varepsilon} dr, (Y_t^{s,x,\varepsilon}, U_t^{s,x,\varepsilon}) \in \partial h, \ \mathbb{E}(\int_0^T h(Y_r^{s,x,\varepsilon})dr) < \infty \end{cases} \tag{1.1}$$

where ϕ is a function of class $\mathcal{C}^2$ with bounded partial derivatives up to 2, $\Theta = \{x : \phi(x) > 0\}$, $\partial\Theta = \{x : \phi(x) = 0\}$, $h : \mathbb{R}^d \longrightarrow (-\infty, +\infty]$ is a proper lower semicontinuous convex function and ∂h is the subdifferential operator.

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- The author proved, as ε goes to 0, the convergence of $(X^{s,x,\varepsilon}, \rho^{s,x,\varepsilon}, Y^{s,x,\varepsilon}, Z^{s,x,\varepsilon}, U^{s,x,\varepsilon})$ solution of system (1.1) to $(\varphi^{s,x}, \rho^{s,x}, \psi^{s,x}, 0, U^{s,x})$ solution of system (1.2).
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LDP

Schilder (1966)

Brownian motion

Freidlin and Wentzell (1984)

SDEs

Rainero (2006)

BSDEs

Essaky (2008)

RBSDEs (Sub-differential operator)

Multivalued BSDEs

LDP

Schilder (1966)

Brownian motion

Freidlin and Wentzell (1984)

SDEs

Rainero (2006)

BSDEs

Essaky (2008)

RBSDEs (Sub-differential operator)

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Outline

1 Introduction

2 Multivalued BSDEs

3 Large deviations

Notations

Inner product and Euclidean norm

$$\langle\, .\, ,\, .\,\rangle \quad |\, .\, | \quad \|z\|^2 = tr(zz^*)$$

Let A be a multivalued operator on $\mathbb{R}^d$

$$D(A) = \{x \in \mathbb{R}^d : A(x) \neq \emptyset\}$$

$$Gr(A) = \{(x, y) \in \mathbb{R}^{2d} : x \in \mathbb{R}^d, y \in A(x)\}$$

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Monotone

$$\langle y_1 - y_2, x_1 - x_2 \rangle \geq 0, \ \forall (x_1, y_1), (x_2, y_2) \in Gr(A)$$

At most one solution

$\forall \lambda > 0, z \in \mathbb{R}^d$, $z \in x + \lambda A(x)$

Maximal monotone

$$(x, y) \in Gr(A) \Leftrightarrow \{\langle y - v, x - u \rangle \geq 0, \forall (u, v) \in Gr(A)\}$$

Uniqueness

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Well-known results about maximal monotone operator

Let $\lambda > 0$.

The single-valued map $J_\lambda = (I + \lambda A)^{-1}$ from $\mathbb{R}^d$ to $\mathbb{R}^d$ is a contraction.

Yosida's approximation

$A_\lambda = \frac{1}{\lambda}(I - J_\lambda)$ is a single-valued map from $\mathbb{R}^d$ to $\mathbb{R}^d$ which is maximal, monotone and Lipschitz continuous with Lipschitz constant $\frac{1}{\lambda}$.

For all $x \in D(A)$, $A(x)$ is a closed convex subset of $\mathbb{R}^d$.
$A^\circ(x) = Proj_{A(x)}(0)$.

For all $x \in D(A)$,

$$|A_\lambda(x)| \leq |A^\circ(x)|, \ \lim_{\lambda \to 0} A_\lambda(x) = A^\circ(x).$$

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Multivalued forward-backward SDE

Let us fix $s \geq 0$ and $x \in \mathbb{R}^d$.

For every $s \leq t \leq T$,

$$\begin{cases} X_t^{s,x,\varepsilon} = x + \int_s^t \beta(X_r^{s,x,\varepsilon})dr + \sqrt{\varepsilon}\int_s^t \sigma(X_r^{s,x,\varepsilon})dW_r \\ Y_t^{s,x,\varepsilon} = g(X_T^{s,x,\varepsilon}) + \int_t^T f(r, X_r^{s,x,\varepsilon}, Y_r^{s,x,\varepsilon}, Z_r^{s,x,\varepsilon})dr - \int_t^T Z_r^{s,x,\varepsilon} dW_r \\ \qquad +K_T^{s,x,\varepsilon} - K_t^{s,x,\varepsilon}, \ \ Y_t^{s,x,\varepsilon} \in \overline{D(A)} \end{cases} \tag{2.1}$$

We interest to the following backward equation

$$\begin{cases} Y_t^{s,x,\varepsilon} = g(X_T^{s,x,\varepsilon}) + \int_t^T f(r, X_r^{s,x,\varepsilon}, Y_r^{s,x,\varepsilon}, Z_r^{s,x,\varepsilon})dr - \int_t^T Z_r^{s,x,\varepsilon} dW_r \\ \qquad +K_T^{s,x,\varepsilon} - K_t^{s,x,\varepsilon}, \ \ Y_t^{s,x,\varepsilon} \in \overline{D(A)}, \ s \leq t \leq T \end{cases} \tag{2.2}$$

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Assumptions

Let $f : [0, T] \times \mathbb{R}^d \times \mathbb{R}^d \times \mathbb{R}^{d\times k} \longrightarrow \mathbb{R}^d$ and $g : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ be continuous functions satisfying the following assumptions :

(A1) g satisfies a Lipschitz condition.

(A2) There exist constants $\mu \in \mathbb{R}$, $K > 0$ such that

- $\forall t, \forall (x, x'), \forall y, \forall (z, z'),$
$$|f(t, x, y, z) - f(t, x', y, z')| \leq K(|x - x'| + ||z - z'||)$$
- $\forall t, \forall x, \forall (y, y'), \forall z,$
$$\langle y - y', f(t, x, y, z) - f(t, x, y', z)\rangle \leq \mu |y - y'|^2$$

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Assumptions

(A3) There exists constant $c > 0$ such that $\forall t, \forall x, \forall y, \forall z$,

$$|g(x)| + |f(t,x,y,z)| \leq c(1 + |x| + |y| + ||z||)$$

(A4) A the multivalued operator satisfies

- $Int(D(A)) \neq \emptyset$, $g(x) \in \overline{D(A)}$,
- $\forall x \in D(A)$, $|A^\circ(x)| \leq \delta(1 + |x|), \delta > 0$.

Solution

Definition

A solution of (2.2) is a triple of progressively measurable processes $\{(Y_t^{s,x,\varepsilon}, Z_t^{s,x,\varepsilon}, K_t^{s,x,\varepsilon}) : s \leq t \leq T\}$ with values in $\mathbb{R}^d \times \mathbb{R}^{d\times k} \times \mathbb{R}^d$, such that

1. $$\mathbb{E}\left(\sup_{s\leq t\leq T} |Y_t^{s,x,\varepsilon}|^2 + \int_s^T ||Z_t^{s,x,\varepsilon}||^2 dt\right) < +\infty,$$
2. $K^{s,x,\varepsilon}$ is continuous and has bounded variation with $K_s^{s,x,\varepsilon} = 0$ a.s.,
3. $Y^{s,x,\varepsilon}$ is continuous and takes values in $\overline{D(A)}$,
4. For any optional process (ν, υ) with values in $Gr(A)$, the measure $\langle Y_r^{s,x,\varepsilon} - \nu_r, dK_r^{s,x,\varepsilon} + \upsilon_r dr\rangle$ is almost surely negative on $[s, T]$.

N'zi and Ouknine (1997) $\Rightarrow$ Existence and Uniqueness

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N'zi and Ouknine (1997) $\Rightarrow$ Existence and Uniqueness

Problem

For every $s \leq t \leq T$,

$$\begin{cases} \varphi_t^{s,x} = x + \int_s^t \beta(\varphi_r^{s,x})dr \\ \psi_t^{s,x} = g(\varphi_T^{s,x}) + \int_t^T f(r, \varphi_r^{s,x}, \psi_r^{s,x}, 0)dr + K_T^{s,x} - K_t^{s,x} \\ \psi_t^{s,x} \in \overline{D(A)} \end{cases} \tag{2.3}$$

Our aim

- Convergence
- LDP

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$$\left\{ \begin{array}{l} \varphi_t^{s,x} = x + \int_s^t \beta(\varphi_r^{s,x})dr \\ \psi_t^{s,x} = g(\varphi_T^{s,x}) + \int_t^T f(r, \varphi_r^{s,x}, \psi_r^{s,x}, 0)dr + K_T^{s,x} - K_t^{s,x} \\ \psi_t^{s,x} \in \overline{D(A)} \end{array} \right. \tag{2.3}$$

Our aim

- Convergence
- LDP

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- Convergence
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Outline

1 Introduction

2 Multivalued BSDEs

3 Large deviations

Large deviations

Definition

Let χ be a topological space. Let $Y^\varepsilon = (Y_t^\varepsilon, s \le t \le T)$ be a family of processes which depends on a parameter ε.

1. A rate function is a lower semicontinuous function Λ defined on χ with values in $[0, +\infty]$. (i.e for all $\alpha \ge 0$, $\{x \in \chi : \Lambda(x) \le \alpha\}$ is a closed subset of χ). A good rate function.
2. $Y^\varepsilon = (Y_t^\varepsilon, s \le t \le T)$ is said to satisfy a LDP with a good rate function Λ if the following condition hold for every Borel set $\mathcal{A} \subseteq \mathcal{C}([s, T], \mathbb{R}^d)$

$$\liminf_{\varepsilon \to 0} \varepsilon \ln(\mathbb{P}(Y^\varepsilon \in \mathcal{A})) \ge - \inf_{\Psi \in \mathring{\mathcal{A}}} \Lambda(\Psi)$$

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Contraction principle

Varadhan (1984)

Let χ and Υ be Polish spaces and let $F^{\varepsilon} : \chi \to \Upsilon$, $\forall \varepsilon > 0$, be continuous maps. Assume F^{ε} converges, as ε goes to 0, to a continuous map F^0, uniformly on each compact subset of χ.
Let $\Lambda : \chi \longrightarrow [0, +\infty]$ be a good rate function.
For all $y \in \Upsilon$, set $\Pi(y) = \inf\{\Lambda(x) : x \in \chi, y = F^0(x)\}$, with the usual convention $\inf \emptyset = +\infty$.
Then, Π is a good rate function on Υ.
Moreover, if Λ is associated to a LDP for a family of measures $(\mu^{\varepsilon})_{\varepsilon>0}$ on χ when ε tends to 0, then $(\mu^{\varepsilon}(F^{\varepsilon})^{-1})_{\varepsilon>0}$ satisfies a LDP on Υ with rate function Π.

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Convergence for backward equation

Theorem

For any $\varepsilon \in]0,1]$ and all $x \in \mathbb{R}^d$, there exists a constant $C > 0$, independent of (s, x, ε), such that

$$\mathbb{E}\left(\sup_{s\leq t\leq T}|Y_t^{s,x,\varepsilon} - \psi_t^{s,x}|^2 + \frac{1}{2}\int_s^T \|Z_r^{s,x,\varepsilon}\|^2 dr\right)$$
$$\leq \quad C\mathbb{E}\left(|X_T^{s,x,\varepsilon} - \varphi_T^{s,x}|^2 + \int_s^T |X_r^{s,x,\varepsilon} - \varphi_r^{s,x}|^2 dr\right)$$

Corollary

$\varepsilon \in]0,1]$, $(s,x) \in [0,T] \times \mathcal{K}$, where $\mathcal{K}$ is a compact subset of $\mathbb{R}^d$.

$$\mathbb{E}\left(\sup_{s\leq t\leq T}|Y_t^{s,x,\varepsilon} - \psi_t^{s,x}|^2\right) \leq C\varepsilon \qquad (3.1)$$

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Cépa (1994)

Lemma of monotony

Skip details

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Let A be a multivalued maximal monotone operator on $\mathbb{R}^d$. Let (Y, K) and (Y', K') be continuous functions on $\mathbb{R}^+$ with values in $\mathbb{R}^d$, such that

1. K, K' are bounded variation,
2. Y, Y' are with values in $\overline{D(A)}$,
3. the measures

$$\langle Y_r - \nu_r, dK_r + \upsilon_r dr \rangle \text{ and } \langle Y'_r - \nu_r, dK'_r + \upsilon_r dr \rangle$$

are almost surely negative on $\mathbb{R}^+$ for each pair of continuous functions (ν, υ) satisfying

$$(\nu_r, \upsilon_r) \in Gr(A), \forall r \in \mathbb{R}^+.$$

Then $\langle Y_r - Y'_r, dK_r - dK'_r \rangle$ is negative on $\mathbb{R}^+$.

Proof of Theorem

Applying Itô's formula to $|Y_t^{s,x,\varepsilon} - \psi_t^{s,x}|^2$

More details

Proof of Theorem

Using Lemma of monotony, assumptions (A1)-(A3), Young's inequality, $2uv \leq \lambda u^2 + \frac{v^2}{\lambda}$ for $\lambda > 0$

▸ More details

Proof of Theorem

Taking $\lambda \geq 2$ such that $2\mu + (1+\lambda)K^2 \geq 0$, by Gronwall's lemma

More details

Proof of Theorem

Therefore, it follows from the Burkhölder-Davis-Gundy inequality that

$$\begin{aligned}
&\mathbb{E}\left(\sup_{s\leq t\leq T}|Y_t^{s,x,\varepsilon}-\psi_t^{s,x}|^2+\frac{1}{2}\int_s^T\|Z_r^{s,x,\varepsilon}\|^2dr\right)\\
\leq\quad & C\mathbb{E}\left(|X_T^{s,x,\varepsilon}-\varphi_T^{s,x}|^2+\int_s^T|X_r^{s,x,\varepsilon}-\varphi_r^{s,x}|^2dr\right).
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■

Large deviations for backward equation

We want to prove that the process $Y^{s,x,\varepsilon}$ satisfies, as ε goes to 0, a LDP.

Definition

Let $(Y_t^{s,x,\varepsilon}, s \leq t \leq T)$ and $(\psi_t^{s,x}, s \leq t \leq T)$ be the solutions of the backward equations of systems (2.1) and (2.3) respectively. Let $\varepsilon > 0$, we define the continuous functions u^ε, u^0 with values in $\overline{D(A)}$ by

$$u^\varepsilon(t,x) \triangleq Y_t^{t,x,\varepsilon}, \; t \in [0,T], \; x \in \mathbb{R}^d$$

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Let $s \in [0, T]$ and $\varepsilon \geq 0$, we define the following applications :

$$F^{\varepsilon}(\phi) \triangleq [t \longmapsto u^{\varepsilon}(t, \phi_t)], \ t \in [s, T], \ \phi \in \mathcal{C}([s, T], \mathbb{R}^d)$$

Proposition

$$Y^{s,x,\varepsilon} = F^{\varepsilon}(X^{s,x,\varepsilon}), \ \psi^{s,x} = F^{0}(\varphi^{s,x})$$

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Proof of Proposition

We use a property of the SDE and the uniqueness of the solution of the Multivalued BSDE.

More details

LDP for Multivalued BSDEs

Definition

Let Λ be the rate function associated to a large deviation principle for a family of processes $(X^{s,x,\varepsilon})_{\varepsilon>0}$. For any $\Phi \in \mathcal{C}([s,T], \overline{D(A)})$, we set

$$\Pi(\Phi) \triangleq \inf \left\{ \Lambda(\Psi) \middle| \, \Phi_t = F^0(\Psi)(t) = u^0(t, \Psi_t), \forall t \in [s,T] \right\}$$

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To apply the contraction principle, we need to prove the following Lemmas

Lemma 1

For any $\varepsilon > 0$, F^{ε} is continuous.

Lemma 2

F^{ε} converges uniformly to F^{0} on every compact subset of $\mathcal{C}([s, T], \mathbb{R}^{d})$.

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Proof of Lemma 1

Let $(\phi^n)_n$ be a sequence in $\mathcal{C}([s,T],\mathbb{R}^d)$, ϕ its limit for the uniform norm, and $\zeta > 0$.

More details

Proof of Lemma 1

Since u^{ε} is continuous, it follows that u^{ε} is uniformly continuous on any compact subset of $[s, T] \times \overline{B}(0, M)$.

More details

Proof of Lemma 2

Let $\mathcal{K}$ be a compact subset of $\mathcal{C}([s, T], \mathbb{R}^d)$, and $\phi \in \mathcal{K}$.

▸ More details

Proof of Lemma 2

Since ϕ is continuous, it follows that $\mathcal{L} = \{\phi_r : \phi \in \mathcal{K}, r \in [s, T]\}$ is a compact subset of $\mathbb{R}^d$.

More details

Proof of Theorem

In view of Lemmas 1 and 2, the proof of theorem is completed by virtue of the contraction principle. ∎

Proof of Theorem

In view of Lemmas 1 and 2, the proof of theorem is completed by virtue of the contraction principle. ■

END

Thanks

Thank you very much ! ! !

Return

Proof of Theorem

Applying Itô's formula to $|Y_t^{s,x,\varepsilon} - \psi_t^{s,x}|^2$

$$
\begin{aligned}
& |Y_t^{s,x,\varepsilon} - \psi_t^{s,x}|^2 + \int_t^T \|Z_r^{s,x,\varepsilon}\|^2 dr \\
= \; & |g(X_T^{s,x,\varepsilon}) - g(\varphi_T^{s,x})|^2 \\
& +2\int_t^T \langle Y_r^{s,x,\varepsilon} - \psi_r^{s,x}, f(r, X_r^{s,x,\varepsilon}, Y_r^{s,x,\varepsilon}, Z_r^{s,x,\varepsilon}) - f(r, \varphi_r^{s,x}, \psi_r^{s,x}, 0)\rangle dr \\
& -2\int_t^T \langle Y_r^{s,x,\varepsilon} - \psi_r^{s,x}, Z_r^{s,x,\varepsilon} dW_r\rangle \\
& +2\int_t^T \langle Y_r^{s,x,\varepsilon} - \psi_r^{s,x}, dK_r^{s,x,\varepsilon} - d\mathcal{K}_r^{s,x}\rangle.
\end{aligned}
$$

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Proof of Theorem

Using Lemma of monotony, assumptions (A1)-(A3), Young's inequality, $2uv \leq \lambda u^2 + \frac{v^2}{\lambda}$ for $\lambda > 0$

$$
\begin{aligned}
& \mathbb{E}\left(|Y_t^{s,x,\varepsilon} - \psi_t^{s,x}|^2 + \int_t^T \|Z_r^{s,x,\varepsilon}\|^2 dr \right) \\
\leq \quad & \mathbb{E}\left(|g(X_T^{s,x,\varepsilon}) - g(\varphi_T^{s,x})|^2 \right) \\
& + \mathbb{E}\left(\int_t^T \left((2\mu + (1+\lambda)K^2)|Y_r^{s,x,\varepsilon} - \psi_r^{s,x}|^2 + |X_r^{s,x,\varepsilon} - \varphi_r^{s,x}|^2 \right) dr \right) \\
& + \mathbb{E}\left(\int_t^T \frac{1}{\lambda} \|Z_r^{s,x,\varepsilon}\|^2 dr \right).
\end{aligned}
$$

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Proof of Theorem

Taking $\lambda \geq 2$ such that $2\mu + (1+\lambda)K^2 \geq 0$, by Gronwall's lemma

$$
\begin{aligned}
& \sup_{s\leq t\leq T} \mathbb{E}\left(|Y_t^{s,x,\varepsilon} - \psi_t^{s,x}|^2\right) + \frac{1}{2}\mathbb{E}\left(\int_s^T \|Z_r^{s,x,\varepsilon}\|^2 dr\right) \\
\leq \; & C\mathbb{E}\left(|X_T^{s,x,\varepsilon} - \varphi_T^{s,x}|^2\right) \\
& + C\mathbb{E}\left(\int_s^T |X_r^{s,x,\varepsilon} - \varphi_r^{s,x}|^2 dr\right).
\end{aligned}
$$

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Proof of Proposition

We use a property of the SDE and the uniqueness of the solution of the Multivalued BSDE.

$$Y_r^{s,x,\varepsilon} = Y_r^{t,X_t^{s,x,\varepsilon},\varepsilon}, \; s \leq t \leq r \leq T$$

Taking $r = t$, we deduce that

$$Y_t^{s,x,\varepsilon} = u^\varepsilon(t, X_t^{s,x,\varepsilon})$$

■

Return

Proof of Lemma 1

Let $(\phi^n)_n$ be a sequence in $\mathcal{C}([s,T],\mathbb{R}^d)$, ϕ its limit for the uniform norm, and $\zeta > 0$.

Then, there exists $M > 0$ such that for all $n \in \mathbb{N}$,

$$\|\phi^n\|_\infty \leq M, \ \|\phi\|_\infty \leq M$$

where

$$\|\theta\|_\infty \triangleq \sup_{r\in[s,T]} |\theta_r|, \ \forall\theta \in \mathcal{C}([s,T],\mathbb{R}^d).$$

Return

Proof of Lemma 1

Since u^ε is continuous, it follows that u^ε is uniformly continuous on any compact subset of $[s, T] \times \overline{B}(0, M)$.

There exists $\eta > 0$ such that $|r - r'| < \eta$ and $|z - z'| < \eta$, $z, z' \in \overline{B}(0, M)$ imply

$$|u^\varepsilon(r, z) - u^\varepsilon(r', z')| \leq \zeta$$

There exists n_0 such that $\forall\ n \geq n_0$, $\|\phi^n - \phi\|_\infty \leq \eta$. Therefore,

$$|u^\varepsilon(r, \phi_r^n) - u^\varepsilon(r, \phi_r)| \leq \zeta$$

■

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Proof of Lemma 2

Let $\mathcal{K}$ be a compact subset of $\mathcal{C}([s,T],\mathbb{R}^d)$, and $\phi \in \mathcal{K}$.

$$\begin{aligned}
\|F^\varepsilon(\phi) - F^0(\phi)\|_\infty &= \sup_{r\in[s,T]} |F^\varepsilon(\phi)(r) - F^0(\phi)(r)| \\
&= \sup_{r\in[s,T]} |u^\varepsilon(r,\phi_r) - u^0(r,\phi_r)| \\
&= \sup_{r\in[s,T]} |Y_r^{r,\phi_r,\varepsilon} - \psi_r^{r,\phi_r}|.
\end{aligned}$$

So,

$$\|F^\varepsilon(\phi) - F^0(\phi)\|_\infty^2 = \sup_{r\in[s,T]} |Y_r^{r,\phi_r,\varepsilon} - \psi_r^{r,\phi_r}|^2.$$

Return

Proof of Lemma 2

Since ϕ is continuous, it follows that $\mathcal{L} = \{\phi_r : \phi \in \mathcal{K}, r \in [s, T]\}$ is a compact subset of $\mathbb{R}^d$.

Thanks to Corollary, there exists a constant C depends only of T and $\mathcal{L}$, such that for every $r \in [s, T]$, all $x \in \mathcal{L}$, all $\varepsilon > 0$, we have

$$\begin{aligned} |Y_r^{r,x,\varepsilon} - \psi_r^{r,x}|^2 &= \mathbb{E}\left(|Y_r^{r,x,\varepsilon} - \psi_r^{r,x}|^2\right) \\ &\leq \mathbb{E}\left(\sup_{t\in[r,T]} |Y_t^{r,x,\varepsilon} - \psi_t^{r,x}|^2\right) \\ &\leq C\varepsilon \end{aligned}$$

Consequently, $\sup_{\phi\in\mathcal{K}} \|F^\varepsilon(\phi) - F^0(\phi)\|_\infty^2 \leq C\varepsilon$. ■

◂ Return